0. Please do the following problems from Chapter 2: 4, 5, 7, 8, 16, 25*, 29, 41, 75, 101-102.

1. **Unitary invariance of the spectrum.** Prove that if \( B = UAU^{-1} \) where \( U \) is unitary, then \( \sigma(B) = \sigma(A) \).

2*. **Spectrum of the lattice laplacian.** Use problem 1 to find the spectrum of the lattice laplacian, defined as follows. The Hilbert space is \( l^2(\mathbb{Z}) \), i.e. the space of functions \( f : \mathbb{Z} \to \mathbb{C} \) such that \( \sum_{j=-\infty}^{\infty} |f(j)|^2 < \infty \) and for such \( f \) we define

\[
(\Delta f)(j) = f(j + 1) + f(j - 1) - 2f(j).
\]

3*. Find the eigenvalues and eigenvectors of Fourier transformation on \( L^2(\mathbb{R}) \).