Assignment 10—extra credit—due May 7

1. Let $M, N$ and $P$ be smooth manifolds; $M$ and $N$ compact and $P$ connected. If $f : M \to N$ and $g : N \to P$ are smooth maps, show that the degree of the composition $g \circ f$ is equal to the product $\deg g \cdot \deg f$.

2. Show that every complex polynomial of degree $n$ extends to a smooth map from $S^2$ to $S^2$ of degree $n$.

3. Let $X$ be a topological space and $f,g$—two continuous maps from $X$ to $S^n$, such that for every $x \in X$ $\|f(x) - g(x)\| < 2$. Prove that $f$ and $g$ are homotopic. If, in addition, $X$ is a smooth manifold and $f$ and $g$ are smooth, prove that they are smoothly homotopic.

4. Let $X$ be a compact manifold. Prove that every continuous map $f : X \to S^n$ can be uniformly approximated by smooth maps. Prove that if two smooth maps $f, g : X \to S^n$ are homotopic, then they are smoothly homotopic.

5. Let $M$ be a smooth manifold of dimension $m$. Prove that if $m < n$, then every smooth map $f : M \to S^n$ is homotopic to a constant.

6. Prove that any smooth map $f : S^n \to S^n$ with degree different from $(-1)^{n+1}$ has a fixed point.