PROMOTING WORK ON EDUCATION IN MATHEMATICS DEPARTMENTS

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1. What Work?

I do not intend to discuss teaching in this article, although it is important and is certainly a sort of work on education. Rather the work I describe is more *research-like* than *teaching-like*. Research-like work produces things or ideas, and goes through the three phases of *discovery*, *publication*, and *review*. When we look at such work, we look at the published result rather than at the process that produced it. There are exceptions to this—cases where we care about the individual contribution in a collaboration, or want to know if an idea was plagiarized from an unpublished manuscript—but by and large the *opus* is the published work.

On the other hand, teaching-like work is a process, the daily exercising of one's knowledge and expertise. We look at such work by observing the process and assigning credit for the changes it brings about. No matter how many brilliant students come out of my classes, you wouldn't credit me with good teaching without knowing to what extent I was responsible for their brilliance, and in order to do that you would have to observe the process itself. There is no *opus*.

We evaluate the two sorts of work differently. Teaching-like work is evaluated continuously, through student surveys, for example. It is also evaluated locally, through comparisons within one institution. Research-like work, on the other hand, is evaluated at discrete intervals (the 6 years between promotions, for example) and globally, through national and international comparisons of published papers. Since research-like work is evaluated in a wider arena, it is possible to rise higher in that line of work; you can become a national treasure rather than simply a local one. This might in part explain the greater glamour attached to research-like work, which often results in cases of mistaken valuation, where, for example, the most meager example of a published paper is deemed more valuable than the most excellent example of teaching.

However, the problem of how to balance the value of teaching and research is not the problem I want to discuss here. Rather, I want to talk about a type of research-like work that often goes unnoticed, and even more often unevaluated, because it is neither teaching nor proving theorems: I want to talk about mathematicians' work on education that is research-like, both in the sense that it feels more like doing mathematics than like teaching, and in the sense that it proceeds through the three phases of discovery, publication, and review. Before looking at some examples, it is worth reviewing those phases in more detail.

Discovery. We believe when we prove theorems that we find out new things. Even if newer things eclipse our work, we believe there is a thread in the fabric of knowledge that is ours. Discoveries may turn out to be re-discoveries or adaptations; however,

there is some element of newness to them, some idea that knowledge is being advanced.
Publication. The things we discover are subject to public discussion because we tell our colleagues about them in journals and books. Our theorems are not idiosyncratic discoveries that we keep to ourselves, our own particular mathematics that works for us. They are refereed and published; they are admired or ignored. Mathematical knowledge is not personal; it is possible to talk about it being correct or incorrect.
Review. There is a culture of professional review surrounding our work as mathematicians. Works, once published, are reviewed in Math Reviews and Zentralblatt, discussed in seminars. Mistakes are found, better methods developed, generalizations proved. By repeatedly passing through this process with success or failure, we are credited with a certain level of expertise by our colleagues. If our expertise is great, we are listened to and our voices carry authority.

2. Examples

2.1. **The Case Studies Project, Boston College.** Solomon Friedberg, an active research mathematician at Boston College, has developed case studies for training graduate teaching assistants. The case studies, written by a group composed largely of research mathematicians, are fictional stories about teaching, intended as sources for group discussion about mathematical and pedagogical questions.

One of the cases is about Daniel, an advanced graduate student, who is teaching a section of Calculus I. After the first examination is handed back a disgruntled student, Sam, comes to Daniel's office. Sam believes that he has been unfairly penalized because he did not receive full credit for a question that asked him to use the definition of the derivative to find the slope of a graph at a certain point. Instead of using the definition, Sam used the power rule, which had not yet been taught in class.

Among the key questions the case brings up are why one teaches precise definitions of mathematical concepts to students, how one communicates those reasons, and how one tests understanding of the definitions. These are thorny questions in curriculum decisions about freshman calculus, because students often come into the course already knowing the rules of differentiation, and are impatient with being required to compute derivatives from first principles. Furthermore, the underlying concept of a limit is often obscured by a preoccupation with the algebraic manipulations required to use the limit definition.

The case was drafted originally by a mathematician, tested in training sessions by mathematicians, and refined in discussions among mathematicians. Many mathematicians feel strongly that it is important for students to understand and use the limit definition. Writing a case about these issues is mathematicians' work because only they can speak about the standards and cultural values of the profession, and balance them within the pedagogical context, in a way that will be listened to by the intended audience for these cases: mathematics graduate students.

The case studies have been published in the Conference Board of the Mathematical Sciences series *Issues in Mathematics Education* under the joint auspices of the American Mathematical Society and the Mathematical Association of America. The following paragraph from the review of this publication in Zentralblatt fur Mathematik by Ubiritan D'Ambrosio of São Paulo shows that the publication is regarded by the mathematics research community as within that community, and not as coming from the community of mathematics education researchers:

> This volume is a good illustration of the different perspectives of mathematicians who teach and of educators with a mathematical formation. Indeed, the former are mathematicians who use the opportunity of having a number of students whose career depends on taking the required courses, to convey the mathematics established in the programs. The latter are educators who see themselves possessing a specific specialty, in the case mathematics, that can be useful in furthering a broad concept of creativity, which are the ideals of students with varied interest, motivation and background. The posture, hence the resulting practices, are not the same. This book is a valuable companion to the former.

2.2. **WeBWorK, University of Rochester.** WeBWorK is an online homework system developed at the University of Rochester, which gives students instant feedback on the correctness of practice problems, and provides automatic grading of homework assignments. It can generate personalized variations on homework problems, allowing instructors to have students work in groups but still be required to work out their own answers. It does not give hints when students make repeated errors (although this would be possible) but rather sends the students to seek the help of a human being.

I asked one of the designers of the system, Michael Gage, whether he thought the fact that the system was designed by mathematicians made it particularly different from other such systems. Here was his reply

> The strong focus on being able to adapt the system to the problems you want to ask, rather than the problems you can ask, stems from a research/educator's point of view more than a math education researcher's (which is legitimately somewhat different) or a programmer's. I doubt that a group from instructional technology would have designed a system with the same set of strengths (and weaknesses).

An example of a "problem you want to ask" is one which asks you for the derivative at a specific value of a mystery function f. You can ask the browser to give you any values you like of the function f, but you cannot see a formula for the function. Thus you are forced to use (and understand the use of) the difference quotient to approximate the derivative.

2.3. **Business Mathematics, University of Arizona.** Business Mathematics is a two-semester course for prospective business majors, developed in collaboration between Richard Thompson of the mathematics department and Chris Lamoureux of the finance department. The students work on four large projects over the two semesters, each of which requires them to make a major business decision, and calls on both mathematical and technological tools. Students work in groups, and present their decisions to the class, as if to a client, using Excel and PowerPoint. Each group has a different set of data to work with, following the same general pattern as a project worked out in class by the professor.

One of the projects, about pricing stock options, arose out of a suggestion by Chris Lamoureux, who didn't realize that it was out of the question to consider translating material on stochastic differential equations to the level of a business calculus course. Students (and instructors!) learn about European versus American options, about puts and calls. They download data on a particular stock from the web, and model option value with simulations using a spreadsheet program. By simulating many runs of the stock market, they come up with a price for their stock option, which agrees quite closely with that given by the Black-Scholes formula.

Of course, there are no stochastic differential equations in this course, nor any treatment of the Black-Scholes formula. However, the sophisticated mathematics and economics behind the Black-Scholes formula were the inspiration for this project, and constructing a project faithful to the mathematics and the economics was necessarily the work of professional mathematicians and economists.

2.4. **The PROMYS program, Boston University.** Mathematician Glenn Stevens, inspired by his experiences as a high school student in Arnold Ross's Secondary Science Training Program at The Ohio State University, started the Program in Mathematics for Young Scientists at Boston University in 1979. The 6 week summer program introduces high school students to the excitement of mathematics research through challenging problems in number theory. More recently PROMYS entered into a collaboration with the Educational Development Center to run a PROMYS for Teachers program, that, in addition to the summer component, has follow-up workshops during the year, which connect the summer immersion experience to activities in the classroom.

Although one can imagine such a program being well run by many different sorts of people, there are two benefits of having an active research mathematician in charge. First, it is in the nature of number theory that bright high school students can come up with questions that penetrate to the very forefront of research, and only someone working at the forefront can give informed answers to such questions. Second, the program should give kids a sense of the deep intellectual excitement of discovering new mathematics; no-one is better equipped than someone with first-hand knowledge of that excitement. Thus, the work of running PROMYS is quintessentially mathematicians' work.

2.5. **Teaching Elementary Teachers, University of Georgia.** Many departments have developed mathematics courses for prospective school teachers. Here is an example from Sybilla Beckmann, who teaches a course for elementary school teachers at the University of Georgia.

We want a story problem that will help students understand why the "invert and multiply" rule for dividing fractions works:

$$\frac{a/b}{c/d} = \left(\frac{a}{b}\right)\left(\frac{d}{c}\right).$$

Here is Sybilla Beckmann's suggestion:

> **Question** If it took 2/3 of a package of wild flower seeds to plant 3/4 of your wild flower garden, then how many packages will it take for the whole garden?
>
> **Answer** On the one hand, the answer is $(2/3)/(3/4)$. On the other hand, thinking about the garden, you have filled 3 parts of it, so you get the fraction of a package of wild flower seeds you

need for each part by dividing 2/3 by 3. Then you need to fill 4 of those parts, so you now multiply by 4. Thus

$$\frac{2/3}{3/4} = \left(\frac{2}{3}\right)\left(\frac{4}{3}\right)$$

What struck me about this example was not its pedagogical qualities (which, indeed, I am not qualified to judge) but its mathematical qualities. It is an example of what mathematicians do all the time. We grapple with an abstract theorem (such as the invert-and-multiply theorem) and try to come to terms with it by constructing a key illuminating example that, on the one hand, is sufficiently complicated so that the theorem is necessary (not $(3/2)/(1/2)$, for example) and, on the other hand, provides an opportunity to "work out" the proof of the theorem by working out the example.

2.6. **And Many More.** Tom Rishel of Cornell University has worked on case studies for training graduate teaching assistants. John Orr of the University of Nebraska has developed his own on-line homework system, e-Grade. Many mathematicians have written textbooks for teacher education courses: David Gay and Fred Stevenson at the University of Arizona; Tom Parker and Scott Baldridge at Michigan State University; Hung-Hsi Wu at UC Berkeley; Sybilla Beckmann at the University of Georgia; and there is a book by Zalman Usiskin, Anthony Peressini, Elena Marchisotto, and Dick Stanley, two of whom are mathematicians. Another example of mathematicians' work on education is the work on calculus reform in the 90s; the list is too long to give here. Mathematicians have been involved in reviewing state standards and frameworks or helping to write them. Robert Case at Northeastern University has set up an effective outreach program to inner city schools in Boston that has introduced AP Calculus classes to many of those schools for the first time, and sent inner city Boston students to Harvard and MIT for the first time.

3. But is it Research?

There are aspects to mathematics research that set it apart from the sort of work I have described: there is its precision and rigor, its foundational role in science, its beauty and depth. I do not claim that the examples I have given possess all these other properties. In mathematics research we place a high value on originality; elegant new proofs of old theorems are not valued as highly as inelegant proofs of new theorems. In work on education we might value things differently, placing a high value on refinement of existing work rather than originality for its own sake (in fact, an irrational attachment to innovation can lead to bad "theorems" in education).

Despite these differences, I claim that there is much work by mathematicians on education that is more research-like than teaching-like, not only because of the superficial resemblance in the way the work proceeds—discovery, publication, and review—but because the questions that arise in this work are mathematical questions, and in order to be able to do the work well you need to be able to think like a mathematician; to have a mathematician's appreciation of abstraction, definition, and proof. I am not saying that that is all you need; you also need an understanding of how students work and think. But the mathematical ingredient is essential. Furthermore, evaluating this work and evaluating teaching are separate activities. There is no reason why being a good teacher should be correlated with doing good work on education. I don't know Michael Gage at all, but I have no reason to know whether he is a good teacher or a bad one simply from the fact that he designs good web-based homework problems. They are different sorts of work.

So how do we evaluate such work?

4. The SEPTC Guidelines at the University of Arizona

The University of Arizona has a long tradition of work on mathematics education, and by the mid-80s had faculty active in many areas: technology in the classroom, outreach to local schools, and developing curricula and textbooks for teacher preparation courses. At that time it made a senior hire (Steven Willoughby) to foster the growth of a mathematics education group. The environment in the department and Willoughby's advocacy led in the early 90s to the development of guidelines for tenuring and promoting faculty whose work was in K-12 mathematics educations. These guidelines provide for a Science Education Promotion and Tenure Committee (SEPTC) at the College level, which works in parallel with the normal college committee in providing information to the Dean and Provost. The guidelines cover all three traditional areas of research, service, and teaching. Note however, the caveat in the introduction:

> Traditional categories (research, teaching, service) may be inappropriate for evaluating science and mathematics educators because the lines between the categories are often blurred. If these categories are to be used, however, caution must be exercised to avoid assigning creative scholarly work to the service or teaching category (where it ordinarily receives less weight in the overall process) simply because it is different from traditional research.

Under the heading of Research or Its Creative Equivalent, the guidelines list the following possibilities:

> ...
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> Worthy contributions could include scholarly books that make a significant contribution, textbooks that are substantially different from, and better than, previous textbooks (if any) on a worthy subject, articles in refereed respected journals that describe and advocate better practice or that present research results relating to learning science or mathematics, improved methods and instruments for evaluation, computer software, movie or television productions that enhance education, and so on.
>
> No one person, of course, will make contributions in all of these ways, but any of these activities, and many similar ones, should be thought of as legitimate research or creative activities. The quality and impact of the work must be seen as the important issues.
>
> ...

The guidelines make a distinction between scholarly work on education and service:

> ...
>
> There may appear to be some overlap between "research or its creative equivalent" and "service" as used here. Many of the opportunities to provide service on a national or international level

> may be indicators of a distinguished reputation, and therefore of high quality research and creativity. However, speaking, service, etc., should not be taken as ipso facto evidence of research and creativity. The research and other contributions must be considered directly, and the opportunities for service taken as only one indicator of the quality of that research and creative contribution.

These guidelines have been used to promote three members of the Department of Mathematics, one to associate professor and two to full professor. As a result of this success, the department has hired three new faculty in mathematics education in the last two years. Furthermore, although the initial group promoted were people with Ph. D.'s in mathematics research who had developed an interest in mathematics education, more recent hires have been faculty whose work is squarely in mathematics education research. This opens up a new possibility of collaborative projects within the department between mathematicians and mathematics education researchers.

5. Collaboration with mathematics education researchers

So far I have talked about work that mathematicians do in their own house: work that not only makes use of mathematician's knowledge but is also rightly under their care. There is another sort of work, where the mathematician consults on a project conceived in another discipline and carried out according to that discipline's norms.

A recent example of such work is the Learning Mathematics for Teaching (LMT) project at the University of Michigan, under the direction of Deborah Ball and Heather Hill, which aims to develop measures of teachers' content knowledge for teaching mathematics. This is different from simply measuring their mathematical knowledge, because it aims to measure whether teachers can use that knowledge for the specific tasks that come up in teaching (much as we might measure whether an engineer can use mathematics to solve engineering problems). One goal of the research is to identify whether there is mathematical knowledge specific to teaching, what that knowledge is, and how to measure it. The project has developed multiple choice questions that could be used, for example, to assess the effectiveness of teacher training programs.

As part of this work, mathematicians have been invited to try out these questions, comment on them, and write their own. This serves a couple of important purposes; calibrating the mathematical soundness of the items and finding out whether there are items that mathematicians have difficulty with but teachers don't (which would point the way to profession-specific mathematics knowledge for teachers).

There is another valuable aspect of such collaborative work. Collaborative efforts between mathematicians and mathematics educators are sometimes hampered by a general lack of mutual respect between the two fields. Therefore, efforts such as LMT are models for how mathematicians and mathematics educators can work productively together. However, to truly serve as a model such work must be valued in mathematics departments, and before that can happen we must learn to value work more domestic to mathematics departments. We can't expect our colleagues to be pioneers in forging connections with other departments if there is not a safe and welcoming environment for them to return to from their voyages of discovery.

We currently do not have such an environment profession-wide for those who work on mathematics education within their own departments. The SEPTC guidelines provide an important model, but such guidelines are not widespread, and even the SEPTC guidelines are limited to mathematicians working in K–12 education.

Furthermore, collaborative work centered in other disciplines, unlike work within mathematics departments, does not suffer from contention about how to evaluate it. Because such work constitutes proper research in another discipline, there is a readily available culture of review in that discipline within which it can be evaluated. We may have trouble persuading mathematics departments to *value* such work, but the question of how to *evaluate* it is easier to answer than the question of how to evaluate mathematicians' work on education in their own departments.

6. Conclusion

An important aspect of the culture of professional review is that it allows us to judge small advances in the field, to put them in the context of great goals. Most research, let's face it, consists of such small advances; however, we can judge when someone is "on the right track" or "working on important problems". In fact, we have as many words for judging mathematical work as the proverbial (and, unfortunately, apocryphal) words for snow in the language of the Eskimos: settling a long-standing conjecture, discovering a major new technique, generalizing a theorem (a mild generalization, perhaps), filling in gaps in the literature, proving a lemma or a theorem, forging deep connections between algebra and geometry, working a well-tilled field. We know how to write and interpret letters of recommendation; we know how to figure out who to ask for letters of recommendation; we know how to judge the quality of a curriculum vitae. We have built up this refined system because we habitually publish our work and review the work of others, and because we regard it as part of our professional practice to do so.

It is time to start developing a similar culture of review for mathematicians' work on education. There is plenty of such work around for us to start talking about, taking seriously, and making it part of our professional lives to evaluate and promote.