Homework 10

1 Derive the row-column rule of matrix product from the definition constructed from compositions of linear transformations, i.e. show that $AB = \begin{bmatrix} A\vec{b_1} & A\vec{b_2} & \cdots & A\vec{b_p} \end{bmatrix} \Rightarrow AB = \sum_{k=1}^{n} a_{ik}b_{kj}$ if $A = \begin{bmatrix} a_{ij} \end{bmatrix}$ is $m \times n$, $B = \begin{bmatrix} b_{ij} \end{bmatrix}$ is $n \times p$.

2 Decide whether the following statement is true or false. Prove it if it's true or give a counter-example if it's false:

For any $n \times n$ matrix A, all entries of A^2 are non-negative.

3 Show that for $\vec{x} \in \mathbb{R}^n$, if $(\vec{x}^T)\vec{x} = 0$ then $\vec{x} = 0$. Use this to show that if $A^T A = 0$, then $A = 0$ where A is an $n \times n$ matrix.