Homework 12

1 Suppose (x_1, y_1) and (x_2, y_2) are two distinct points on the line $y = mx + b$. Show that the vectors $\begin{bmatrix} x \\ x_1 \\ x_2 \end{bmatrix}$, $\begin{bmatrix} y \\ y_1 \\ y_2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ are linearly dependent. Use this to show that the equation of the line can be described by $\det \begin{bmatrix} 1 & x & y \\ 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \end{bmatrix} = 0$.

2 Use the permutation definition of determinant to show that $\det(A) = \det(A^T)$ where A is an $n \times n$ matrix.

3 Show that $\det(A^{-1}) = (\det(A))^{-1}$.

4 $f_1(x)$, $f_2(x)$ and $f_3(x)$ are three given functions. Show that if there are non-zero real numbers a_1, a_2 and a_3 such that $a_1 f_1(x) + a_2 f_2(x) + a_3 f_3(x) = 0$ for all x, then $\det \begin{bmatrix} f_1''(x) & f_1'(x) & f_1(x) \\ f_2''(x) & f_2'(x) & f_2(x) \\ f_3''(x) & f_3'(x) & f_3(x) \end{bmatrix} = 0$ for all x.