Homework 13

1 Consider $V = \mathbb{R}^+ = \{x > 0\}$ with vector addition $\oplus$ and scalar multiplication $\odot$ defined by

$$x \oplus y = xy \text{ and } k \odot x = x^k \text{ for any } x, y \in V \text{ and any scalar } k.$$

Show that $(V, \oplus, \odot)$ is a vector space.

2 Let $H = \{p \in \mathbb{P}_n | p(1) = 0\}$ where $\mathbb{P}_n$ is the vector space consisting of all polynomial of degree $\leq n$. Show that H is a subspace of $\mathbb{P}_n$.

3 Suppose H and S are subspaces of a vector space V. Show that $H \cap S$ is a subspace of V.

4 Suppose $\text{Nul}(A) = \{\vec{0}\}$ for some $m \times n$ matrix A. Show that the matrix transformation $\vec{x} \mapsto A\vec{x}$ is one-to-one.