Homework 16 – cover lectures through 4/15

1 Let $\vec{u} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \vec{v} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ and $W = \text{span}(\vec{u}, \vec{v})$. Consider a point $p = (0, 0, 1)$. Compute the distance between p and the plane W by following these steps: Let $\vec{p} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. In parts (a) and (b), we shall find the vector $\vec{x}$ joining p and the plane W and $\vec{x}$ is orthogonal to W. I strongly recommend that you draw pictures to demonstrate the ideas.

a) Let $\vec{q} \in W$ and $\vec{x} = \vec{p} - \vec{q}$. Express $\vec{q}$ in terms of $\vec{u}$ and $\vec{v}$ and use this to describe $\vec{x}$.

b) Suppose $\vec{x} \perp W$, then $\vec{x}$ is perpendicular to any vector in W, in particular to $\vec{u}$ and $\vec{v}$. Use this condition to solve for the undetermined constants in the description of $\vec{x}$.

c) Use $\vec{x}$ to find the desired distance (i.e. distance between p and W).

2 A simple model for the weather says that a sunny day is 90% likely to be followed by another sunny day, and a rainy day is 50% likely to be followed by another rainy day. Let $\vec{x}_k = \begin{bmatrix} s_k \\ r_k \end{bmatrix}$ where s_k is the probability that the k-th day is sunny and r_k is the probability that the k-th day is rainy. The weather is then modeled by the dynamical system $\vec{x}_{k+1} = A\vec{x}_k$ where A is the transition matrix. Suppose we know that initially it is a sunny day, i.e. $\vec{x}_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

a) Find $\vec{x}_k$.

b) What is the long term weather prediction?

3 Let $\vec{w}_1 = \begin{bmatrix} 1 \\ 3 \\ -2 \\ 0 \\ 2 \\ 0 \end{bmatrix}, \vec{w}_2 = \begin{bmatrix} 2 \\ 6 \\ -5 \\ -2 \\ 4 \\ -3 \end{bmatrix}, \vec{w}_3 = \begin{bmatrix} 0 \\ 0 \\ 5 \\ 10 \\ 0 \\ 15 \end{bmatrix}, \vec{w}_4 = \begin{bmatrix} 2 \\ 6 \\ 0 \\ 8 \\ 4 \\ 18 \end{bmatrix}$ and $W = \text{span}(\vec{w}_1, \vec{w}_2, \vec{w}_3, \vec{w}_4)$. Find a basis for $W^{\perp}$.

Hint: Construct a matrix A with $\vec{w}_1^T, \vec{w}_2^T, \vec{w}_3^T, \vec{w}_4^T$ be its rows. What is the connection between W and a special subspace related to A^T, and what is the known orthogonal complement to that special subspace?