Review for Test 2

This test covers sections 2.1-2.3, 3.1, 3.2, 4.1-part of 4.4. Although chapter 1 is not tested explicitly, it is still necessary to know the theory in $\mathbb{R}^n$ well.
The test will be similar to the first test in format. There will be definitions, computations, true/false with justification and proofs. Definitions must be stated precisely. A definition is not a theorem. A definition is an adopted fact, not a consequence. For example, the definition of a square matrix A being invertible involves the existence of a matrix B of the same size such at $AB = BA = I$. Telling me the strategy to solve for an inverse matrix is NOT stating the definition. Even uniqueness of B and hence its special name A^{-1} is a consequence and not part of the definition. When proving a general statement, assuming examples (such as the size of the matrix) is not acceptable.
The material for this test is more conceptual than it is computational. Make sure you did all the assignments even those not required to be turned in. Many problems that are computational still requires justification and more "proof oriented". Look at the problems in the exercises.
In chapter one, matrices play a passive role to represent a linear system by storing coefficients and the numbers on "the right hand side of the equations". Even matrix vector product is defined as a linear combinations of the columns with the entries of the vector serving as weights. In chapter 2 and 3, matrices take central stage.

- Important definitions: elementary matrices, matrix product (constructed from composition of linear transformation), transpose matrix, inverse matrix, inverse linear transformation, determinants (using cofactor expansions and permutations).
- Important skills (partial list):
  - matrix algebra in particular non-commutativity of matrix product, transpose and inverse of matrix product.
  - using inverse matrix to solve linear system.
  - using the equivalent statements in invertible matrix theorem, including implications about determinants.
  - computing determinants by cofactor expansions, permutations and through row reduction by elementary row operations.
  - properties of determinants.
  - connection of matrix theory to properties of linear transformations in $\mathbb{R}^n$, e.g. invertibility, composition of linear transformation to matrix products.

Chapter 4 covers the basic theory of general vector spaces. Although $\mathbb{R}^n$ remains an important example, you should be familiar with more abstract vector spaces like matrix space, function spaces, polynomial space. On these vector spaces, a linear transformation is not a matrix transformation.

- Important definitions: vector space, subspace (+ null space, column space of a matrix), linear combination, linear independence, span, spanning set, basis, coordinate vector, linear transformation.

- Important skills (partial list):
  - Verify vector space/ subspaces.
  - Determine linear independence relation for vectors in general vector spaces.
  - Determine whether a set of vectors is a spanning set or a basis.
  - Describe $\mathrm{Nul}(A)$, $\mathrm{Col}(A)$.
  - Find a basis for $\mathrm{Nul}(A)$, $\mathrm{Col}(A)$.
  - Find the coordinate vector with respect to a given basis.
  - Find the kernel and range of a linear transformation.

Any proof type problems covered in lecture, quizzes, homework (and assignments) can be directly relevant to the test. When you are asked to prove a statement of a known theorem, do not quote the theorem itself as a proof. Your proof must be coherent and self contained. You can quote some known results but the role they play in your proof must be fully explained.