Review for Test 3

This test covers sections 4.4–4.5, 5.1–part of 5.4, 5.6 – 5.7, 6.1–6.4. As before, prior chapters though not explicitly tested are still essential as tools for these latter sections. As always, lecture notes, assignments, written homework, quizzes are all relevant, *even if* they are not in the text. The textbook is only part of the resources.

Test format will be similar to previous tests, and possibly includes definitions, computations, true/false with justification and proofs.

The main point of the remainder of chapter 4 involves mapping from a general vector space to $\mathbb{R}^n$ by the isomorphism given by the coordinate mapping determined by a chosen basis. 4.5 supplies the concept of dimension of a subspace/ vector space that gives an alternative way to check for a basis.

- Important definitions: basis, coordinate mapping, dimension of a vector space, change of coordinate matrix.
- Important skills (partial list):
  - Find coordinate vector for a vector relative to a basis
  - Find change of basis matrix
  - Find matrix representation of a linear transformation in general vector space under the coordinate mapping (this material and exercises are oddly placed in part of section 5.4)
  - Find dimension of $\mathrm{Col}(A), \mathrm{Nul}(A)$ and other subspaces.
  - Know how to map a general vector space onto $\mathbb{R}^n$ to solve problems, e.g. linear combination, linear independence, existence and uniqueness of solutions to linear systems etc.

Chapter 5 introduces the concepts of eigen-system and gives a glimpse of the power of these concepts in solving a vast area of application problems. Eigenvectors are special directions where the effect of the associated matrix transformation is simply a scaling by a number called eigenvalue. Eigenvectors are also where the effect of the matrix transformation is the most exagerated. A linear system $\vec{y} = A\vec{x}$ where $\vec{x}, \vec{y}$ are defined as appropriate to respective application problems, can be studied via understanding eigenvalues and eigenvectors. We restrict our attention to real systems with real eigenvalues and eigenvectors.

- Important definitions: eigenvalue, eigenvector, eigen-space, eigen-basis, similar matrices, diagonalizable.
- Important skills (partial list):
  - Determine if a number/ vector s an eigenvalue/ eigenvector of a matrix.
  - Find the characteristic polynomial/ equation and eigenvalues of a matrix.
  - Find eigenvalues of a triangular matrix.
  - Algebraic/ geometric multiplicities of an eigenvalue.
  - Find a basis for an eigen-space.

- Diagonalizes a matrix $A$.
- Compute high powers of a diagonalizable matrix.
- Verify statements involving similarity transformation/ similar matrices.
- Solving discrete and continuous linear dynamical sytems.
- Specifying a fixed point is attracting, repelling or a saddle point

Chapter 6 ventures into orthogonality relations, which requires the structure of an inner product, which induces a norm which in turns induces a metric/ distance. Each of these operators have useful repercussion in geometric descriptions of vectors. One of the main points of orthogonality is orthogonal projections, that gives the best approximation of a vector on a subspace. This gives the natural notion of decomposing a vector into a sum of its orthogonal projections onto a subspace $W$ and its orthogonal complement $W^{\perp}$. When a basis is orthogonal, it allows us a convenient description of the orthogonal projection of a vector onto a subspace $W$. This leads to the construction of an orthogonal/ orthonomal basis using Gram-Schmidt.

- Important defintions: inner product, dot product, norm, distance/ metric, orthogonal/ orthonormal vectors/ set, orthogonal/ orthonormal basis, orthogonal projection, orthogonal complement, orthogonal decomposition, orthogonal matrix.
- Important skills (partial list):
  - Know the properties of inner product, norm, metric and how to verify them.
  - Compute length of a vector, distance between two vectors, angle between two vectors.
  - Normalize a vector.
  - Check a set for orthogonality directly or using a matrix by computing $A^T A$ where $A$ contains the set of vectors in question.
  - Compute the orthogonal projection of a vector onto another vector or on a subspace.
  - Decompose a vector into a component in a subspace $W$ and its orthogonal complement $W^{\perp}$.
  - Use orthogonal projections to obtain estimates on a subspace.
  - Know the special orthogonal complements involving column and null space of a matrix and how to describe those subspaces, e.g. in vector form, by specifying a basis etc.
  - Describe any vector in a subspace with an orthogonal basis in terms of a linear combination of the basis vectors.
  - Generate an orthogonal basis from any given basis.