\[ \sum_{n=0}^{\infty} \frac{a_n}{(n+1)(n+2)(n+3)} x^n = \int_0^x \left( \frac{8}{x} - 2x^2 + 4x - 2 \right) dx = 0 \]

\[ x = r = 1, \quad P = 1 \]

Let us suppose the Cauchy–Hadamard criterion.

\[ \lim_{n \to \infty} \left| \frac{a_n}{a_{n+1}} \right| = \gamma \]

\[ a_n = \text{Uniform} \]

Since

\[ \lim_{n \to \infty} a_n = 0 \]
Thus we set

\[ G_{n+2} = \frac{2}{n+2} G_n \]

First solution is obtained in \( n \) are even

\( n = 2k \)

\[ a_{2(k+1)} = \frac{1}{k+1} a_{2k} \]

\[ y = 1 + x^2 + \frac{x^4}{2} + \frac{x^6}{2 \cdot 3} + \cdots = e^{x^2} \]

One can check that equation (x) can be integrated

\[ y'' - 2\frac{\partial^3}{\partial x^3} x y = 0 \]

\[ y' - 2xy = c \]

If \( c = 0 \)

\[ y = e^{x^2} \]

If \( c = 1 \)

\[ y = e^{x^2} \int_0^x e^{-t^2} \, dt \]

In this case all \( n \) are even or odd

\( a_1 = 1 \quad a_2 = \frac{2}{3} \quad a_5 = \frac{2}{3} \cdot \frac{2}{5} \)
\[ \frac{\partial^2 u}{\partial r^2} + \frac{2}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = \frac{\partial^2 u}{\partial t^2} \]

(1)

Assuming that

\[ \frac{\partial^2 u}{\partial t^2} = -c^2 \frac{\partial^2 u}{\partial r^2} c \frac{\partial^2 u}{\partial \theta^2} \]

The equation can be rewritten as follows:

Looking for solutions in the form

\[ u(r, \theta, t) = R(r) \Theta(\theta) T(t) \]

One finds the indicial equation

\[ s^2 - s + 1 = 0 \]

\[ s = \frac{1 \pm \sqrt{5}}{2} \]

The solution of the equation is

\[ a_{2k+1} \]

\[ \frac{a_{2k+1}}{k} = \frac{1}{3} \]
\( (x-r)^y \frac{dy}{dx} \cdot \frac{x}{e^x} = \frac{2x-1}{x} = \frac{1}{r} \)

\( (1-x^2) y' - \frac{y}{x} = 0 \)

\( x = 0 \) so \( y = 0 \) gives equation (*). We seek the general solution.

\( z^2 + n = z + x + y \)

\( \left( z^2 + n + x \right) y = z \) where \( n = \frac{x^2}{2} \).

We consider cases according to \( n \) and \( x \).

\( n = 0 \) then \( x \neq 0 \) so \( z \neq 0 \).

\( z = \frac{2x}{e^x} \)

\( \frac{2x}{e^x} = \frac{2e}{x} \)

\( \frac{2x}{e^x} = \frac{2e}{x} \frac{1}{e^x} \)

\( \frac{2x}{e^x} = \frac{2e}{x} \frac{2e}{c} \)
\[
\begin{align*}
    z &= c \int_0^x \frac{\sqrt{1-x^2}}{\sqrt{x}} \, dx = \ln \sin x \\
    &= \frac{\sqrt{1-x^2}}{x} \\
    \frac{2}{z} &= \frac{2}{x} (1-x^2) + \ln c \\
    y &= -\frac{2}{x} - \frac{2}{x} (1-x^2) + \ln c
\end{align*}
\]