\[ y' + \frac{2}{3} \frac{y}{x} y'' = 0 \]

Let us present a solution in the form

\[ y = \frac{x^p}{x^q} \]

Homework:

Math 422

Z. Zouhary

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Equations
\[
\sum_{k=1}^{\infty} \frac{k}{4^k} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots
\]

We seek a form of power series.

\[
F = 1 + \frac{z}{2} \frac{d}{dz} F + \frac{z^2}{2!} \frac{d^2}{dz^2} F + \cdots = 0
\]

is known as a Frobenius series.

\[
F = \sum_{n=0}^{\infty} a_n z^n
\]

This is equation (2).

\[
F = \sum_{n=0}^{\infty} \left( \frac{z^n}{n!} \right)
\]

Based on Frobenius series.

\[
F = 0
\]

Thus, \( F = 0 \) on \( r_1 \).
We use\[ s_{s-1} = \frac{x^2}{e^s - s^2} \]

Thus, continuous \( x \) and satisfy\[ \frac{16}{(k+1)^2} = 1 \]

And

\[ \frac{16}{(k+1)^2} = \frac{(k+1)(n+1)}{(k+1)(n+1)} \]

\[ \frac{16}{(k+1)^2} \]

Thus we must satisfy following conditions

\[ x = 1 \]
$0 \quad \Phi$

There is a general formula

$$y = \frac{1}{\sqrt{2 \pi}} \int_{-\infty}^{\infty} e^{-x^2/2} \, dx$$

This is also true

$$\frac{d}{dx} e^{x^2} = 2xe^{x^2}$$

Hence

$$\frac{d}{dx} e^{x^2} = 2xe^{x^2} + x^2 e^{x^2}$$
\[
\frac{\partial^n p(x)}{\partial t^n} = \frac{1}{\eta^n} \sum_{k=0}^{\eta^n} \frac{k! (\eta^n-k)!}{(\eta^n-z)^k} \left( \frac{z^n-x^n}{z^n} \right)^{k-1} \left( \frac{z^n-x^n}{z^n} \right) \frac{d^k}{dx^k} \left[ \frac{1}{(z^n-x^n)^{k-1}} \right]
\]

where

\[
\psi(x) = \frac{1}{\eta^n} \sum_{k=0}^{\eta^n} \frac{k! (\eta^n-k)!}{(\eta^n-z)^k} \left( \frac{z^n-x^n}{z^n} \right)^{k-1} \left( \frac{z^n-x^n}{z^n} \right) \frac{d^k}{dx^k} \left[ \frac{1}{(z^n-x^n)^{k-1}} \right]
\]

By the use the binomial expansion:

\[
\psi(x) = \frac{1}{\eta^n} \sum_{k=0}^{\eta^n} \frac{k! (\eta^n-k)!}{(\eta^n-z)^k} \left( \frac{z^n-x^n}{z^n} \right)^{k-1} \left( \frac{z^n-x^n}{z^n} \right) \frac{d^k}{dx^k} \left[ \frac{1}{(z^n-x^n)^{k-1}} \right]
\]

Because

\[
\frac{d^k}{dx^k} \left[ \frac{1}{(z^n-x^n)^{k-1}} \right] = \frac{1}{(z^n-x^n)^{k-1}} \frac{d^k}{dx^k} \frac{1}{(z^n-x^n)^{k-1}}
\]

In proof:

\[
\frac{d}{dx} \left[ \frac{1}{(z^n-x^n)^{k-1}} \right] = \frac{1}{(z^n-x^n)^{k-1}} \frac{d}{dx} \left[ \frac{1}{(z^n-x^n)^{k-1}} \right]
\]

Finally:

\[
\psi(x) = \frac{1}{\eta^n} \sum_{k=0}^{\eta^n} \frac{k! (\eta^n-k)!}{(\eta^n-z)^k} \left( \frac{z^n-x^n}{z^n} \right)^{k-1} \left( \frac{z^n-x^n}{z^n} \right) \frac{d^k}{dx^k} \left[ \frac{1}{(z^n-x^n)^{k-1}} \right]
\]
Problem 6

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Homework N 2.

Following equality is valid:

\[ x y_1^2(x) + y_0^2(x) = \left( \frac{d}{dx} \left( \frac{1}{y_1(x)} \right) \right)^2 \]

Instead:

\[ \frac{d}{dx} \left( \frac{1}{y_1(x)} \right)^2 \]

Then:

\[ x y_0^2(x) + y_1^2(x) = y_1(x) \]

Now replace expression in the paranthesis is cancelled:

\[ x y_0(x) + y_1(x) = y_1(x) \]
\[ \frac{1}{2} (h_2^2 + h_1^2) + x = \frac{1}{2} \int x \, dx \quad \text{Kron} \]

\[ \frac{\partial}{\partial x} \left( \int x \, dx \right) = x \, \frac{\partial}{\partial x} \left( \int x \, dx \right) \]

By applying the set of equations:

\[ x = \left( x \, h_2^2 + h_1^2 \right) \]

Hence

\[ \frac{x}{h_1} \frac{x}{h_2} = \frac{h_2}{h_1} \frac{x}{h_2} = \frac{h_2}{h_1} \]

However

\[ \frac{h_2}{h_1} x = \frac{h_2}{h_1} \int x \, dx = \frac{h_2}{h_1} \left[ \frac{x}{2} \right] \]

\[ = \left( \frac{h_2}{h_1} \right) \frac{x}{2} + \left( \frac{h_2}{h_1} \right) \frac{x}{2} \]

Let us study the expression.

Homework 6:

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